Experimental study of dynamic behaviors and routes to chaos in DC–DC boost converters

D. Cafagna *, G. Grassi

*Dipartimento di Ingegneria dell’Innovazione, Università degli Studi di Lecce, via Monteroni, 73100 Lecce, Italy

Accepted 23 November 2004

Abstract

This paper illustrates an experimental study of a current-programmed DC–DC boost converter, with the aim of investigating possible pathways through which the converter may enter chaos. In particular, based on experimental measurements, it is shown that variations of input voltage and reference current can generate periodic, subharmonic, quasi-periodic and chaotic behaviors.

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1. Introduction

In nonlinear circuits and systems a great variety of strange phenomena has been observed, including subharmonics, quasi-periodic oscillations, chaotic behaviors, intermingled basins of attraction, synchronization properties and multi-scroll attractors [1–6]. These behaviors have been intensively studied in the cross-disciplinary science of chaos. In particular, it has been recently observed that a large number of power electronic circuits can exhibit deterministic chaos [7–9]. Referring to power DC–DC converters, some investigations have shown that buck, boost and buck-boost converters are prone to subharmonic behavior and chaos [10–13]. Even though most of the approaches proposed until now are very interesting, they mainly present theoretical or simulated results. As a consequence, there is a lack of experimental analysis on the parameter domains for which chaotic behavior may occur. Therefore, this paper aims to bridge this gap by presenting an experimental study of some dynamic phenomena that can occur in current-programmed boost converters. In particular, the paper illustrates a novel hardware implementation able to show some pathways through which the boost converter may enter chaos. The manuscript is organized as follows. In Section 2 the state equations of the boost converter are briefly summarized. Moreover, the circuit implementation of the proposed current-programmed converter is illustrated in detail. In Section 3 it is experimentally shown that variations of the reference current can lead to interesting routes to chaos. Analogously, the experimental study carried out in Section 4 highlights that the implemented converter may enter chaos by considering the input voltage as a bifurcation parameter.
2. Hardware implementation

This section describes the hardware implementation of the proposed current-programmed boost converter, which is constituted by a power stage and a control circuit (Fig. 1). The proposed power stage includes an inductor $L$, a diode $D$, a DC source $V_{in}$, a switch $S$, a load resistance $R$, a capacitor $C$ and resistors $R_1, R_2, R_5$. The converter is assumed to operate in continuous conduction mode, that is, the inductor current $i(t)$ never falls to zero [14]. Hence, there are two switch states:

(i) switch $S$ ON and diode $D$ OFF;  (ii) switch $S$ OFF and diode $D$ ON.

The two states toggle periodically, that is, the boost takes state (i) for $nT \leq t < (n + d)T$ and state (ii) for $(n + d)T \leq t < (n + 1)T$, where $T$ is the switching period, $d$ is the duty cycle and $n$ is an integer. Therefore, the state equations of the converter are

$$
\begin{align*}
\frac{di(t)}{dt} & = \frac{1}{RC} \frac{dv(t)}{dt} - \frac{(R_1 + R_2 + R_5)}{L} i(t) + \frac{1}{L} V_{in} \quad \text{for} \ nT \leq t < (n + d)T;
\end{align*}
$$

Fig. 1. Experimental current-programmed boost converter.
\[
\begin{bmatrix}
\frac{dv(t)}{dt} \\
\frac{di(t)}{dt}
\end{bmatrix} = \begin{bmatrix}
-1/RC & 1/C \\
-1/L & -(R_1 + R_2)/L
\end{bmatrix}\begin{bmatrix}
v(t) \\
i(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
1/L
\end{bmatrix}V_{in} \quad \text{for} \quad (n + T) \leq t < (n + d)T,
\]

where \(v(t)\) is the voltage across the capacitor. The inductor current \(i(t)\) is chosen as the programming variable, which generates the ON-OFF driving signal for the switch \(S\) after the comparison with a reference current \(I_{\text{ref}}\) where \(I_{\text{ref}} = V_{\text{ref}}/R_5\) (see Fig. 1). While \(S\) is ON, \(i(t)\) increases until reaches the value of \(I_{\text{ref}}\). Then, \(S\) is turned OFF and remains OFF until the next cycle begins. Note that the proposed boost implementation does not include a voltage feedback [12].

The switch \(S\) in the power stage is realized using a MOSFET, whereas its control circuit is based on several analog and digital devices that are described in the following. At first, the OpAmp LM339 is used as a comparator [15]. In particular, the LM339 compares the reference voltage \(V_{\text{ref}} (= R_5 I_{\text{ref}})\) with the voltage across the resistance \(R_5\) which is proportional to the current \(i(t)\) through the inductor \(L\). Therefore, the output of the comparator is high when the inductor current reaches the value \(I_{\text{ref}}\), whereas it is low when the inductor current is less than \(I_{\text{ref}}\).

The generation of the clock signal is now described. In order to generate a square wave with amplitude of 5 V, frequency \(f = 1/T = 10\, \text{kHz}\) and duty cycle \(d = 0.9\), the integrated device NE555C (along with proper resistors and capacitors) is considered [15]. By making the derivative of the rising edge of the square wave, it is possible to obtain an impulsive signal that represents the SET input of the S-R latch [16]. Additionally, by making the derivative of the falling edge, the signal able to control the duty cycle is obtained. Referring to the latch, it has been implemented using two NOT gates and two NAND gates [15]. Its output signal is high (i.e., the MOSFET is ON) when an impulsive signal arrives at the SET input. On the other hand, its output signal is low (i.e., the MOSFET is OFF) when a proper impulsive signal arrives at the RESET input. Such RESET signal, by means of an OR gate, can be either the signal able to control the duty cycle or the output of the comparator. The OR gate has been implemented using a NAND gate along with two NOT gates, whereas the output of the S-R latch drives the gate of the MOSFET by means of a buffer based on the MC34084 OpAmp.

Finally, before analyzing the boost dynamics, it is worth noting that the measurements have been carried out by varying the values of the reference current \(I_{\text{ref}}\) and of the input voltage \(V_{\text{in}}\), whereas the values of the remaining circuit parameters have been chosen as reported in Fig. 1. The measurements have been carried out by using a Protek 6510 oscilloscope and by considering the sensing resistor \(R_1\) in series with the inductor \(L\).

3. Experimental route to chaos by varying the reference current \(I_{\text{ref}}\)

The attention is focused on the control circuit of the current-programmed boost converter (Fig. 1). In particular, the behavior of the boost is analyzed by varying the reference current \(I_{\text{ref}}\), whereas the value of the input voltage has been chosen as \(V_{\text{in}} = 7\, \text{V}\). At first, by choosing the value of the reference current as \(I_{\text{ref}} = 0.49\, \text{A}\), the fundamental periodic operation has been found. To this purpose, Fig. 2(a) shows the corresponding PSpice waveform of the inductor current,
whereas Fig. 2(b) shows the measured inductor current of the implemented converter, confirming the fundamental periodic behavior. When the reference current $I_{ref}$ is increased, other operating regimes can be found. To this purpose, a detailed analysis for slightly different values of $I_{ref}$ has been carried out. For example, Fig. 3(a) shows the PSpice time waveform of the current $i(t)$ for a period-two subharmonic operation, with $I_{ref} = 0.76$ A. Such period-two behavior is confirmed by the experimental time waveform of the current $i(t)$ reported in Fig. 3(b). Additionally, by taking $I_{ref} = 1.03$ A, it is possible to obtain a quasi-4$T$ periodic operation. To this purpose, Fig. 4(a) shows the PSpice waveform of the current $i(t)$, whereas Fig. 4(b) shows the experimental waveform of the inductor current, confirming the quasi-4$T$ periodic behavior. When the value of the current $I_{ref}$ is further increased, the chaotic operating regime is obtained. Namely, Fig. 5(a) illustrates the simulated waveform of the current $i(t)$ for $I_{ref} = 1.05$ A, whereas Fig. 5(b) shows the experimental chaotic waveform of the inductor current.

Fig. 3. 2$T$ subharmonic operation ($I_{ref} = 0.76$ A): (a) time waveform of the inductor current using PSpice; (b) experimental time waveform of the inductor current (vertical scale: 0.2 V/div = 200 mA/div; horizontal scale: 0.1 ms/div).

Fig. 4. Quasi-4$T$ subharmonic operation ($I_{ref} = 1.03$ A): (a) time-domain waveform of the inductor current using PSpice; (b) experimental time waveform of the inductor current (vertical scale: 0.2 V/div = 200 mA/div; horizontal scale: 0.2 ms/div).
Fig. 5. Chaotic operation \((I_{\text{ref}} = 1.05 \text{ A})\): (a) time waveform of the inductor current \(i(t)\) using PSpice; (b) experimental time waveform of the inductor current (vertical scale: 0.2 V/div = 200 mA/div; horizontal scale: 0.2 ms/div).

Fig. 6. FFT spectra of the inductor current under different dynamic behaviors. (a) Fundamental periodic operation \((I_{\text{ref}} = 0.49 \text{ A})\); (b) period-two subharmonic behavior \((I_{\text{ref}} = 0.76 \text{ A})\); (c) quasi-4T periodic operation \((I_{\text{ref}} = 1.03 \text{ A})\); (d) chaotic regime \((I_{\text{ref}} = 1.05 \text{ A})\).
Finally, in order to better illustrate the way the proposed boost changes its dynamic behavior, some FFT spectra have been carried out by taking the previous values of the bifurcation parameter $I_{\text{ref}}$. Namely, Fig. 6 (a) illustrates the FFT spectrum under a fundamental periodic operation ($I_{\text{ref}} = 0.49$ A), whereas Fig. 6 (b) shows the spectrum in the case of a period-two subharmonic behavior ($I_{\text{ref}} = 0.76$ A). Additionally, Fig. 6 (c) reports the FFT spectrum under a quasi-$4T$ periodic operation ($I_{\text{ref}} = 1.03$ A), whereas Fig. 6 (d) illustrates the continuous and broad-band spectrum in the case of the chaotic regime ($I_{\text{ref}} = 1.05$ A).

4. Experimental route to chaos by varying the input voltage $V_{\text{in}}$

Referring to the power stage of the current-programmed boost converter, the dynamics are analyzed by varying the input voltage $V_{\text{in}}$, whereas the value of the reference current has been fixed to $I_{\text{ref}} = 1.05$ A. At first, the fundamental
periodic behavior has been found by choosing the value of the input voltage as $V_{in} = 10 \text{ V}$. To this purpose, Fig. 7(a) shows the PSpice inductor current waveform, whereas Fig. 7(b) shows the measured inductor current of the implemented converter, confirming the period $T$ operation. When the input voltage $V_{in}$ is decreased, many other operating regimes are possible. To this purpose, a detailed analysis for different values of $V_{in}$ has been carried out. For example, Fig. 8(a) shows the PSpice waveform of the current $i(t)$ for a $2T$ subharmonic operation, with $V_{in} = 8 \text{ V}$. Such period-two behavior is confirmed by the experimental time waveform of the inductor current reported in Fig. 8(b). Additionally, by taking $V_{in} = 7.2 \text{ V}$, it is possible to obtain a quasi-$4T$ periodic operation. To this purpose, Fig. 9(a) shows the PSpice waveform of the current $i(t)$, whereas Fig. 9(b) shows experimental waveform of the inductor current, confirming the quasi-$4T$ periodic behavior. When the value of the input voltage is further decreased, the chaotic regime is obtained. Namely, Fig. 10(a) illustrates the simulated waveform of the current $i(t)$ for $V_{in} = 7 \text{ V}$, whereas Fig. 10(b) shows the experimental chaotic waveform of the inductor current.

Fig. 9. Quasi-$4T$ subharmonic operation ($V_{in} = 7.2 \text{ V}$): (a) time waveform of the inductor current using PSpice; (b) experimental time waveform of the inductor current (vertical scale: 0.2 V/div = 200 mA/div; horizontal scale: 0.2 ms/div).

Fig. 10. Chaotic operation ($V_{in} = 7 \text{ V}$): (a) time-domain waveform of the inductor current using PSpice; (b) experimental time waveform of the inductor current (vertical scale: 0.2 V/div = 200 mA/div; horizontal scale: 0.1 ms/div).
Finally, some FFT spectra are reported for the previous values of the bifurcation parameter $V_{\text{in}}$. Namely, Fig. 11(a) illustrates the FFT spectrum under a period-$T$ operation ($V_{\text{in}} = 10$ V), whereas Fig. 11(b) shows the spectrum in the case of a 2$T$ subharmonic behavior ($V_{\text{in}} = 8$ V). Additionally, Fig. 11(c) illustrates the FFT spectrum under a quasi-4$T$ periodic operation ($V_{\text{in}} = 7.2$ V), whereas the continuous and broad-band spectrum reported in Fig. 11(d) highlights the occurrence of the chaotic regime ($V_{\text{in}} = 7$ V).

5. Conclusions

This paper has illustrated an experimental study of some dynamic pathways in a current-programmed boost converter. In particular, it has been demonstrated that variations of the reference current $I_{\text{ref}}$ (related to the control circuit) can generate periodic, subharmonic, quasi-periodic and chaotic behaviors. Analogously, experimental measurements have shown that variations of the input voltage $V_{\text{in}}$ (related to the power stage) can lead to similar routes to chaos. Finally, all the behaviors have been analyzed by considering the FFT spectra of the inductor current.

References